Rankings of Income Distributions:

A Note on Intermediate Inequality Indices

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Abstract

The purpose of this paper is to analyze the advantages and disadvantages of several intermediate inequality measures, paying special attention to the unit-consistency axiom proposed by Zheng (2007). First, we demonstrate why one of the most referenced intermediate indices, proposed by Bossert and Pfingsten (1990), is not unit-consistent. Second, we explain why the invariance criterion proposed by Del Río and Ruiz Castillo (2000), recently generalized by Del Río and Alonso-Villar (2007), leads instead to inequality measures that are unaffected by the currency unit. Third, we show that the intermediate measures proposed by Kolm (1976) may also violate unit-consistency. Finally, we reflect on the concept of intermediateness behind the above notions together with that proposed by Krtscha (1994). Special attention is paid to the geometric interpretations of our results.

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Keywords: Income distribution; Intermediate inequality indices; Unit-consistency

Introduction

There is a wide consensus in the literature about the properties an inequality measure has to satisfy when using it to compare income distributions having the same mean. Basically, we must invoke the symmetry axiom—which guarantees anonymity—and the Pigou-Dalton principle of transfers—which requires a transfer of income from a richer to a poorer person to decrease inequality.[1] However, if we are interested in comparing two income distributions that have different means, we need to specify the type of mean-invariance property we want our inequality indices to satisfy. This requires introducing another judgment value into the analysis, and no agreement has been reached among scholars with respect to this matter. Some opt to invoke the scale invariance axiom, according to which the inequality of a distribution remains unaffected when all incomes increase (or decrease) by the same proportion. This is the approach followed by the relative inequality indices. Others prefer, instead, to call on the translation invariance axiom, under which inequality remains unaltered if all incomes are augmented (or diminished) by the same amount, thereby giving rise to the absolute inequality measures. However, as Kolm (1976) pointed out, some people may prefer an intermediate invariance approach between these two extreme views. He labeled such an inequality attitude as "centrist", against the "rightist" and "leftist" labels he used to term the aforementioned relative and absolute notions, respectively.

So far, the intermediate and absolute inequality indices have rarely been applied to ranking income distributions since these measures are cardinally affected by the currency unit in which incomes are expressed. In a recent paper, Zheng (2007) invoked a new axiom, the unit-consistency axiom, requiring that inequality rankings between income distributions remain unchanged when all incomes are multiplied by a (positive) scalar.[2] In this new scenario, not only relative measures but also absolute and intermediate measures that satisfy the unit-consistency axiom appear as plausible options for empirical research.

The purpose of this paper is to analyze the advantages and disadvantages of several intermediate inequality measures, paying special attention to the unit-consistency axiom. First, we demonstrate why one of the most referenced intermediate indices, proposed by Bossert and Pfingsten (1990) (B-P hereafter), is not unit-consistent. A geometric interpretation of this result is also given. This analysis reveals that the problem lies in the iso-inequality criteria behind that index, which helps to explain why the decomposable intermediate inequality measures à la B-P proposed by Chakravarty and Tyagarupananda (2000) do not satisfy unit-consistency either, as shown by Zheng (2007). Second, we explain both analytically and graphically why the invariance criterion proposed by Del Río and Ruiz Castillo (2000), recently generalized by Del Río and Alonso-Villar (2007), leads instead to inequality measures that are unaffected by the currency unit. Third, we demonstrate that the intermediate measures proposed by Kolm (1976) may also violate unit-consistency. Finally, we reflect on the concept of intermediateness behind the above notions together with the "fair compromise" notion proposed by Krtscha (1994), closely examining their geometric interpretations.

Unit-consistency and intermediate inequality measures

In order to ensure independence of the unit of measurement without imposing scale invariance, Zheng (2007) introduced the following property into inequality measures:[3]

Unit consistency. For any two distributions x, $y \in \Re^n_{++}$ and any inequality measure I ($I : \bigcup_{n\geq 2} \Re^n_{++} \to \Re_+$), if $I(x) < I(y)$, then $I(\theta x) < I(\theta y)$ for any $\theta \in \Re_{++}$.

Certainly, any relative inequality measure satisfies the above property since they are defined as those where $I(\theta x) = I(x)$ for any $\theta \in \Re_{++}$. However, we should keep in mind that there are other unit-consistent indices, apart from the scale invariant ones. In this vein, the family of indices proposed by Zheng (2007), which includes the "fair compromise" measure proposed by Krtscha (1994), offers examples of intermediate indices satisfying this property.[4] In what follows, we analyze several intermediate inequality equivalence criteria by distinguishing between linear and non-linear invariances.

Linear invariance criteria

The μ-inequality concept proposed by B-P is the intermediate inequality measure most frequently mentioned in the literature.[5] According to this invariance criterion, an intermediate inequality index should satisfy the following condition for a given $\mu \in [0,\ 1]$:

$$I_{\mu}[x] = I_{\mu}\left[x + \tau(\mu x + (1-\mu)1^n)\right]$$

for any $n \geq 2, x \in \Re^n_{++}$, and $\tau \in \Re$, such that $x + \tau(\mu x + (1-\mu)1^n) \in \Re^n_{++}$, where $1^n \equiv (\underbrace{1,......,1}_{n})$.

"TAKE IN FIGURE 1"

As shown in Figure 1 for $n = 2$ and $\mu = 0.25$, for a given income distribution $x \in \Re^n_{++}$, the distributions which are μ-inequality equivalent to it are those located on the line defined by point x and vector $\mu x + (1-\mu)1^n$ (which represents an intermediate attitude between the relative ray given by x and total equity given by 1^n).[6] In particular, $I_{\mu}(x) = I_{\mu}(y)$.

By using a numerical example in a five-dimensional space, Zheng (2007) showed that the decomposable intermediate inequality measures à la B-P characterized by Chakravarty and Tyagarupananda (2000) are not unit-consistent. By using the same distributions, we can prove that B-P's index also violates the above axiom.

In fact, if $\mu = 0.5$, $x = (1,2,3,4,5)$ and $y = (0.1, 0.1, 0.2, 0.6, 0.6)$, then $I_{0.5}(x) = 0.068 > 0.015 = I_{0.5}(y)$, but $I_{0.5}(10x) = 0.122 < 0.148 = I_{0.5}(10y)$, where $I_{\mu}(x) = (1+s)\left[1 - \prod_{i=1}^{n}\left(\frac{x_i + s}{\overline{x} + s}\right)^{1/n}\right]$, $s \equiv \frac{1-\mu}{\mu}$ and $\overline{x}$ represents the mean of distribution x.

In Figure 2, we illustrate why this popular intermediate inequality equivalence criterion leads to measures that do not satisfy the unit-consistency axiom. A formal proof is given in the appendix.

Thicker dash lines represent the two μ-invariance lines passing through points x and $2x$, that is, the set of distributions equivalent to x and $2x$, respectively. Vector y represents an income distribution that is equivalent to x, since it is located on the invariance line of the latter. It is easy to see that any distribution between y and z has a larger inequality level than x because of the Pigou-Dalton transfer axiom. However, distributions resulting from doubling their individual incomes (which are located between $2y$ and $2z$) would have instead a lower inequality level than distribution $2x$. Therefore, changes in the currency unit do affect rankings between income distributions.

"TAKE IN FIGURE 2"

The above graphical analysis permits us to illustrate that the aforementioned five-dimensional example was not an isolated one. We have shown that, even in a two-dimension space, for any given income distribution it is possible to find an interval of distributions that violate the axiom when comparing them with the former distribution. The explanation of this behavior relies on the notion of inequality equivalence proposed by B-P. The slope of the inequality invariance line given by direction $\mu x + (1-\mu)1^n$ does depend on the total income of distribution x. In fact, keeping the relative inequality as constant, the larger the total income, the larger this slope (the slope of the invariance line corresponding to $2x$ is larger than that of x, as shown in Figure 2).

This means, first, that μ represents a different intermediate inequality attitude depending on the distribution in which the index is evaluated. Since the invariance lines are, therefore, not parallel, it is impossible to state that μ-inequality rankings are not affected by changes in the scale when comparing any two distributions.[7] Thus, we have shown that the heart of this equivalence criterion is incompatible with the unit-consistency axiom, so that any measure based on this notion violates this axiom.

Second, the μ-inequality concept approaches the "rightist" view of inequality (the invariance line becomes closer to the relative ray) when aggregate income rises (see Figure 2).[8] This means that results obtained by using this intermediate concept can be quite close to those obtained with relative measures, which can be seen as unsuitable for a "centrist" measure.

"TAKE IN FIGURE 3"

Moreover, in Figure 3, which shows the $\mu -$iso-inequality contours corresponding to distribution $x = (20, 80)$ for two μ values, we see that the invariance line corresponding to $\mu = 0.5$ is roughly indistinguishable from the ray passing by x (which defines the iso-inequality line of relative measures). In fact, to obtain an iso-inequality contour closer to the "leftist" view (i.e., closer to the absolute ray), it would be necessary to choose a parameter value extraordinary low (for example $\mu = 0.005$).[9] This suggests that parameter μ, even though it takes a value between 0 and 1, has not a clear economic interpretation, since its value does not give us any idea of the invariance line location. In particular, in the above example, $\mu = 0.5$ does not represent an equidistant position between the relative and absolute rays, but is instead a position close to the "rightist" view.

If one is interested in defining a linear "centrist" measure as a convex combination of a relative and an absolute ray, one could fix not only parameter μ, but also the reference distribution that gives rise to the "rightist" and "leftist" views. In this regard, Del Río and Ruiz-Castillo (2000) (DR-RC hereafter) proposed the (v, π)-inequality, where v is a vector belonging to the n-dimensional simplex, and $\pi \in [0,\ 1]$. The first component fixes the distribution of reference, while the second refers to the convex combination of the relative and absolute rays associated to v.[10] Thus, if we chose a value of π close to 1, the notion represents value judgments rather "rightist", while if π is close to 0, the inequality attitude is rather "leftist", as compared to the distribution of reference.

Once these two components are fixed, we can calculate the n-dimensional simplex vector $\alpha = \pi v + (1-\pi)\left(\frac{1}{n}\right)1^n$ (see Figure 4), which defines the direction of the inequality equivalence ray, and the set of income distributions $\Gamma^{'}(\alpha)$ for which α represents an intermediate attitude.[11] This set can be expressed as follows:

$\Gamma^{'}(\alpha)=\left\{x\in D:\ \pi_x v_x+(1-\pi_x)\frac{1^n}{n}=\alpha,\ \text{for some}\ \pi_x\in[0,\,1]\right\}$, where D is the set of all possible ordered income distributions, and v_x represents the vector of income shares associated to x (it therefore belongs to the n-dimensional simplex). If we want vector α to represent an intermediate notion, it must be Lorenz-dominated by the egalitarian distribution. On the other hand, this vector can only be used for income distributions that are (weakly) Lorenz-dominated by it.

"TAKE IN FIGURE 4"

Thus, an intermediate inequality index is defined as (v, π)-invariant in the set of income distributions $\Gamma^{'}(\alpha)$ if for any $x\in\Gamma^{'}(\alpha)$ the following expression holds:

$$I_{(v,\pi)}(x)=I_{(v,\pi)}(y),\ \text{for any}\ y\in P_{(v,\pi)}(x),$$

where $P_{(v,\pi)}(x)=\left\{y\in D:\ y=x+\tau\left(\pi v+(1-\pi)\frac{1^n}{n}\right),\ \tau\in\Re\right\}$ represents the inequality invariance line. Note that this line is obtained as the convex combination, given by π, of the "leftist" and "rightist" views associated with vector v. On the other hand, since $x\in\Gamma^{'}(\alpha)$, the invariance line can also be obtained as a convex combination of the "leftist" and "rightist" views associated to vector v_x by using π_x.

The distribution of reference, v, plays a very important role in this approach. Note that vector v does not necessarily have to coincide with vector v_x (as shown in Figure 4). However, in comparing any two distributions x and y (which can be assumed to have a higher mean without loss of generality), vector v could be chosen as the income shares of x, i.e., $v=v_x$. By using this benchmark, together with the parameter π reflecting the inequality-invariance value judgments of society, it would be possible to determine whether y has a lower inequality than the distribution reached if π100% of the income gap had been distributed according to income shares in x and $(1-\pi)$100% in equal amounts among individuals. Note that, in doing so, the same vector of reference ($v=v_x$) has to be used for both distributions x and y. It would not be possible to use $v=v_x$ for measuring the inequality level corresponding to x, while using $v = v_y$ in the case of distribution y, since that would imply that different inequality attitudes would be used for each distribution.[12] In other words, once v and π are chosen, they cannot be changed: the same intermediate notion must be used when comparing any two income distributions.[13] Therefore, when studying the evolution of an economy over time, this approach allows the possibility of taking into account the starting point.[14]

As opposed to B-P's approach, the same vector α is now used for obtaining the iso-inequality lines, which implies that the invariance lines passing through distributions x and $2x$ are parallel (see Figure 5). Therefore, the (v, π)-invariance notion does not have the problem shown in Figure 2. In other words, if distributions x and y are in the same invariance line, distributions $2x$ and $2y$ also share a common inequality level (see Figure 5). In the Appendix, we formally prove that according to the (v, π)-invariance concept, inequality rankings are unaffected by the monetary unit in which incomes are expressed. This can help explain why the family of indices based on this approach, proposed by Del Río and Alonso-Villar (2007) (DR-AV hereafter), does satisfy the unit-consistency axiom. The invariance lines corresponding to three of these indices are shown in Figure 3, where $\pi \in \{0.25, 0.5, 0.75\}$ and $v = (0.2, 0.8)$. We can see that $\pi = 0.5$ leads to an iso-inequality contour that is "equidistant" from the "rightist" and "leftist" views of distribution $(20, 80)$, when choosing the vector of reference $v = (0.2, 0.8)$.

"TAKE IN FIGURE 5"

Non-linear invariance criteria

An alternative to the above intermediate notions is to assume that the iso-inequality contours are not straight lines. In this regard, Krtscha (1994) proposes an adaptive intermediate notion that gives rise to parabolas. According to his "fair compromise" notion, to keep inequality unaltered, any extra income should be allocated among individuals in the following way. The first extra dollar of income should be distributed so that 50 cents goes to the individuals in proportion to the initial income shares, and 50 cents goes in equal absolute amounts. The second extra dollar should be allocated in the same manner, starting now from the distribution reached after the first dollar allocation, and so on. According to this invariance notion, two income distributions, x and y, have the same inequality level so long as $y-\bar{y}=\sqrt{\tau}\left(x-\bar{x}\right)$, where $\bar{y}=\tau\bar{x}$ and the bar represents the average of the corresponding distribution.

The "fair compromise" index [and the generalizations proposed by Zheng (2007)] does satisfy unit-consistency and decomposability, as shown by the latter. However, this "centrist" attitude is rather challenging since it approaches the absolute view rather soon when income increases, which makes it difficult for inequality to decrease when analyzing an economy over time. In Figure 6, we can see that, according to Krstcha's index, inequality would remain unaltered with respect to distribution $\left(20,80\right)$ if the poorer reached an income of 400 and the richer of 590, which would imply income shares of 40% and 60%, approximately. If we continued our previous simulation and plotted the invariance curve for larger income levels (which are not shown in Figure 6), we would reach distribution $\left(81920,\ 84367\right)$, which represents income shares of 49.3% and 50.7%, respectively.

This proximity to the absolute view does not contradict, however, the tendency of this index to a relative inequality measure when income increases to infinity, while keeping inequality constant, as shown by Zheng (2004). He proved that when moving repeatedly along an iso-inequality contour, the curve becomes eventually a straight line, so that the intermediate notion becomes relative. This does not mean, however, that the iso-inequality contour is close to the "rightist" view. A relative ray can be as close as wanted to the line representing total equity. In fact, when repeatedly moving along the invariance line, Zheng (2004) showed that $\lim_{k\to\infty}\frac{x_i^{(k+1)}}{x_i^{(k)}}=\tau$. On the other hand, since $x_i^{(k+1)}=\sqrt{\tau}x_i^{(k)}+\left(\tau-\sqrt{\tau}\right)\bar{x}^{(k)}$, then $\frac{x_i^{(k+1)}}{x_i^{(k)}}=\frac{\sqrt{\tau}\left[x_i^{(k)}+\left(\sqrt{\tau}-1\right)\bar{x}^{(k)}\right]}{x_i^{(k)}}$. It is easy to prove that the limit of the above quotient tends to τ if and only if $\lim_{k\to\infty}\frac{\bar{x}^{(k)}}{x_i^{(k)}}=1$. Therefore, when moving repeatedly along the invariance line, distribution $x^{(k)}$ tends to the egalitarian distribution $\overline{x}^{(k)}$.

"TAKE IN FIGURE 6"

Kolm's (1976) "centrist" measures also lead to iso-inequality contours that are not straight lines.[15] However, as opposed to Zheng's family of indices, that of Kolm does not cover the whole intermediate space since, as shown in Figure 6,[16] "centrist" attitudes are close to the "leftist" view, while those near the "rightist" view are not permitted for any parameter value.[17] On the other hand, Kolm's "centrist" measures may violate the unit-consistency axiom when $\xi \neq 0$ (if $\xi = 0$ the index is homogeneous of degree 1 and, therefore, it does satisfy the axiom). In this regard, if $\xi = 10$ and $\varepsilon = 10$, for distributions $x = (2,2,6,7,7)$ and $y = (2,2,3,8,8)$, it follows that $I_{(10,10)}(x) = 1.63 < 1.66 = I_{(10,10)}(y)$ while $I_{(10,10)}(2\,x) = 4.13 > 3.94 = I_{(10,10)}(2y)$, where $I_{(\xi,\varepsilon)}(x) = \overline{x} + \xi - \left[\frac{1}{n} \left(\sum_{i=1}^{n} \left(x_i + \xi \right)^{1-\varepsilon} \right) \right]^{\frac{1}{1-\varepsilon}}$. Therefore, (ξ,ε)-inequality rankings may be affected by currency units.

Final remarks

The unit-consistency axiom, recently invoked by Zheng (2007), requires that inequality rankings between income distributions remain unaffected by the unit in which incomes are expressed. This axiom does not impose such strong value judgments on inequality measurement as the scale invariance condition, and therefore, intermediate indices satisfying it appear to be plausible options for empirical research.

Intermediate measures are especially useful when comparing two income distributions, x and y, where (at the same time) y has a higher absolute inequality level and a lower relative inequality level than x, according to the absolute and the relative Lorenz criterion, respectively.

We have revised the “centrist” measures offered by the literature in order to verify whether they are unit-consistent. We have shown that both the class of intermediate inequality indices proposed by Bossert-Pfingsten (1990) and Kolm (1976) are affected by the currency unit. Therefore, only the “fair compromise” index proposed by Krtscha (1994), the generalizations proposed by Zheng (2007), and the indices proposed by Del Río and Alonso-Villar (2007)—which, as opposed to the others, are ray invariant—are intermediate inequality measures satisfying unit-consistency.

One advantage of both the “fair compromise” index and the family of measures proposed by Zheng (2007) is that they are decomposable, which is helpful for empirical analysis. Krtscha’s index also has a clear economic interpretation, but the flexibility of this approach, imposing an intermediate notion based on marginal changes with respect to each distribution, entails a challenging “centrist” attitude. The fact that the invariance set is a parabola approaching the absolute view makes it rather difficult for inequality to decrease when analyzing an economy over time.

The family of indices proposed by Del Río and Alonso-Villar (2007), which is based on the invariance notion put forward by Del Río and Ruiz-Castillo (2000), also brings a clear economic interpretation of intermediateness while proposing a more conservative approach. Since the iso-inequality contours are straight lines, the “centrist” attitude remains constant when income increases. Therefore, this approach does not allow any change in individuals’ value judgments regarding inequality when varying aggregate income, which seems plausible for analysis in the short and medium run, bringing a complementary perspective to the former.

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Appendix

Result #1. *According to the inequality invariance notion proposed by B-P, inequality rankings are affected by the monetary unit in which incomes are expressed.*

Proof:

Assume that y is an income distribution that belongs to the iso-inequality contour corresponding to x. Therefore, y is located in the plane defined by distributions x and 1^n since it can be written as

$$y = x + \tau_y \left[\mu x + (1-\mu)1^n \right] = x + \tau_y \alpha_x , \tag{A1}$$

where $\alpha_x = \mu x + (1-\mu)1^n$ represents the direction of the invariance line. In what follows, we will show that the unit-consistency axiom is violated by any inequality index based on the inequality invariance notion proposed by B-P, since distribution λy has lower μ-inequality than distribution λx, where $\lambda \in \mathbb{N}$ is higher than 1.

From expression (A1), it follows that distribution λy can be written as

$$\lambda y = \lambda x + \lambda \tau_y \alpha_x = \lambda x + \lambda n \bar{\alpha}_x \tau_y \frac{\alpha_x}{n\bar{\alpha}_x} , \tag{A2}$$

where $\bar{\alpha}_x$ represents the mean of the corresponding distribution and $\frac{\alpha_x}{n\bar{\alpha}_x}$ is a simplex vector.

Let us denote w as the distribution that has the same inequality level as λx and the same mean as λy.[18] The first requirement guarantees that $w = \lambda x + \tau_w \alpha_{\lambda x} = \lambda x + n\bar{\alpha}_{\lambda x} \tau_w \frac{\alpha_{\lambda x}}{n\bar{\alpha}_{\lambda x}}$, where $\alpha_{\lambda x} = \mu \lambda x + (1-\mu)1^n$. By also using the second requirement, we obtain the following:

$$w = \lambda x + \lambda n \bar{\alpha}_x \tau_y \frac{\alpha_{\lambda x}}{n\bar{\alpha}_{\lambda x}} , \tag{A3}$$

since the mean of distribution λy is $\lambda \bar{y} = \lambda \bar{x} + \lambda n \bar{\alpha}_x \tau_y \frac{1}{n}$ and that of distribution w is $\bar{w} = \lambda \bar{x} + n \bar{\alpha}_{\lambda x} \tau_w \frac{1}{n}$.

On the other hand, note that $\lambda\alpha_x = \mu\lambda x + (1-\mu)\lambda 1^n = \alpha_{\lambda x} + (\lambda - 1)(1-\mu)1^n$. Therefore, distribution $\lambda\alpha_x$ Lorenz-dominates distribution $\alpha_{\lambda x}$, which implies that α_x also Lorenz-dominates $\alpha_{\lambda x}$. From expressions A2 and A3, it follows that distribution λy Lorenz-dominates distribution w. Since these two distributions have the same mean, the former must have lower μ-inequality than the latter. Consequently, distribution λy has lower μ-inequality than distribution λx, since w and λx are in the same invariance line. This completes the proof. □

Result #2. *The inequality ranking between two distributions according to the invariance notion proposed by DR-RC remains unaltered when changing the currency unit.*

Proof:

The (v,π)-inequality concept proposed by DR-RC only allows comparisons between income distributions that are in the same plane, which is defined by one of the distributions and the egalitarian distribution.[19] Therefore, let x and y be two income distributions in that plane. First, we assume that x and y are in the same invariance line and show that distributions λx and λy are also inequality equivalent (where $\lambda \in \Re_{++}$). Second, we assume that y has lower inequality than x and demonstrate that λy has lower inequality than λx.

Step 1. First assume that x and y are in the same invariance line. Therefore, $y = x + \tau_y \alpha$, where $\alpha = \pi v + (1-\pi)\left(\frac{1}{n}\right)1^n$. This implies that $\lambda y = \lambda x + \lambda \tau_y \alpha$. In other words, λy is located in the invariance line corresponding to distribution λx since this line can be written as $P_{(v,\pi)}(\lambda x) = \{z \in D:\ z = \lambda x + \tau_z \alpha,\ \tau_z \in \Re\}$.

Step 2. Now assume that distribution y has lower (v,π)-inequality than distribution x. Therefore, there a distribution y^* exists—having the same mean as y and the same (v,π)-inequality as x—which can be obtained from distribution y via regressive transfers. Subsequently, y Lorenz-dominates y^*. Thus, distributions λy and λy^* have the same mean and the former Lorenz-dominates the latter. This implies that λy has lower (v,π)-inequality than λy^*. Since by step 1 λy^* has the same (v,π)-inequality level as λx, it follows that λy has lower (v,π)-inequality than λx, which completes the proof. □

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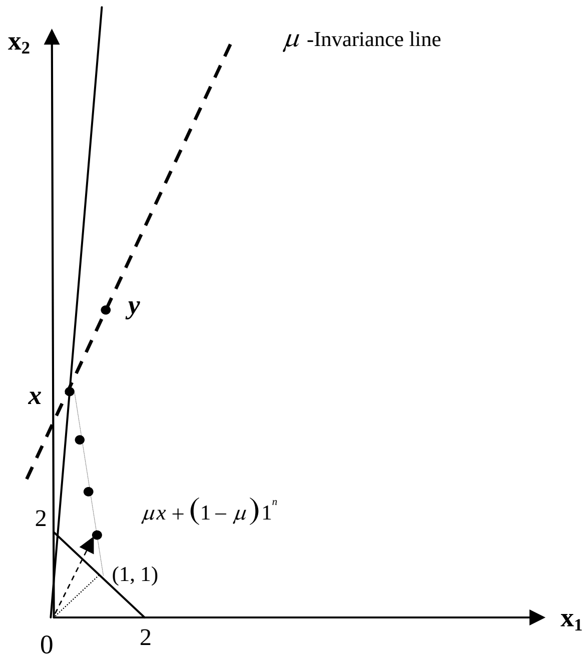

Figure 1. Invariance in B-P **(** $n = 2$, μ=0.25**)**

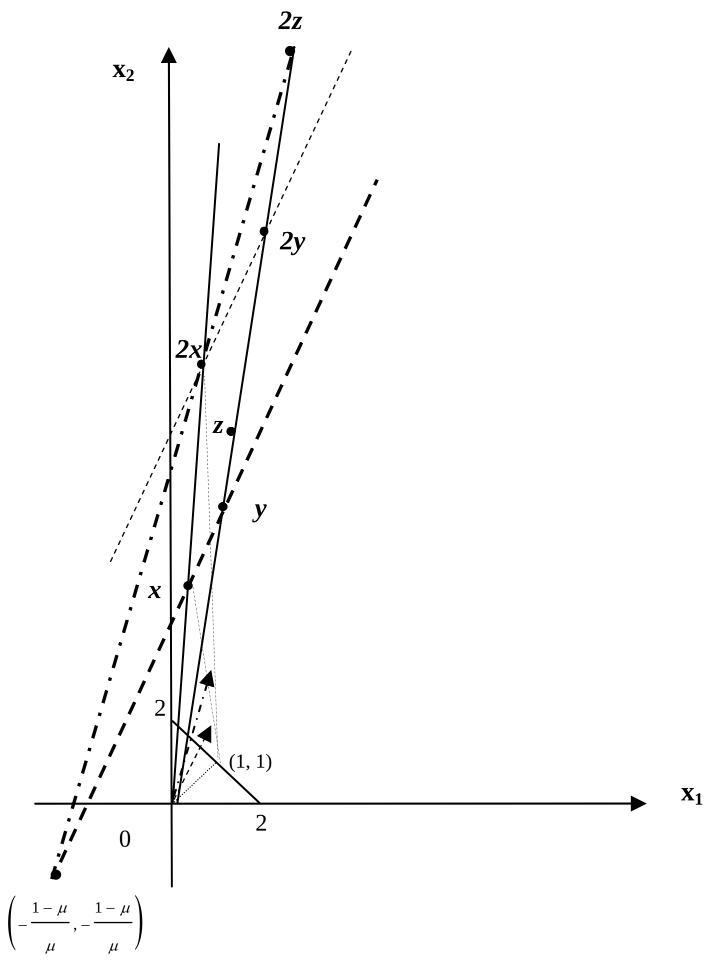

Figure 2. Unit consistency in B-P ($n = 2$, $\mu = 0.25$).

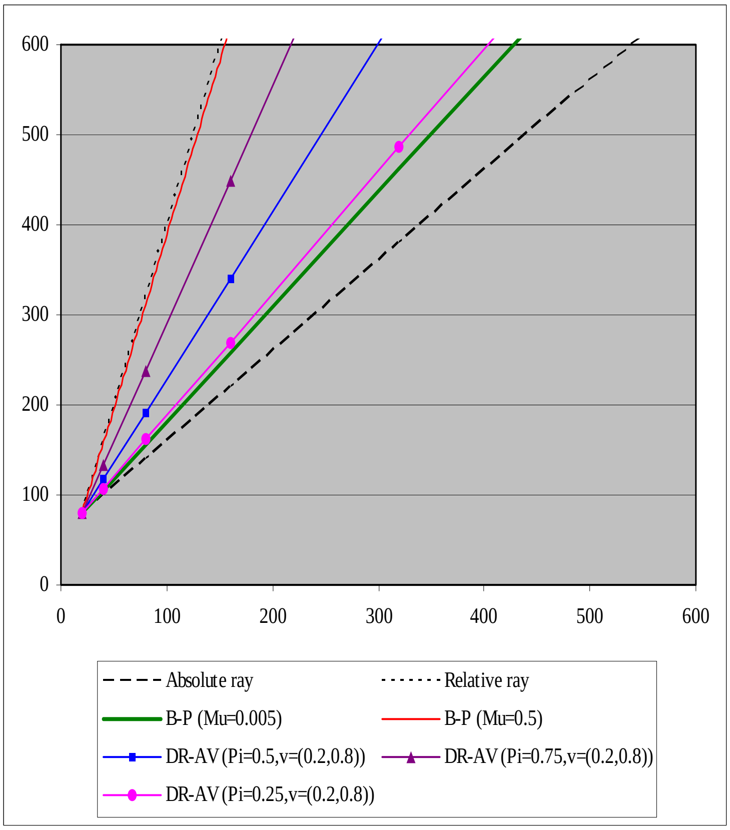

Figure 3. Iso-inequality contours corresponding to distribution $(20,80)$: Linear cases

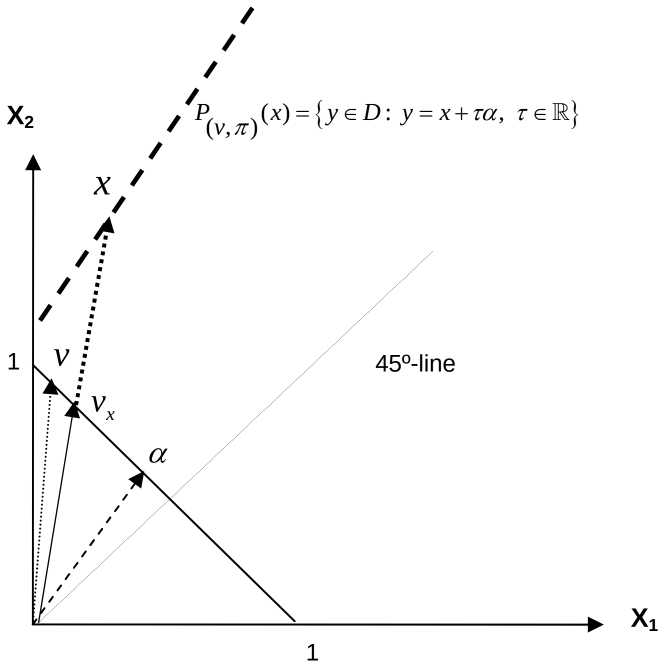

Figure 4. Invariance in DR-RC ($n = 2, \ \pi = 0.25$).

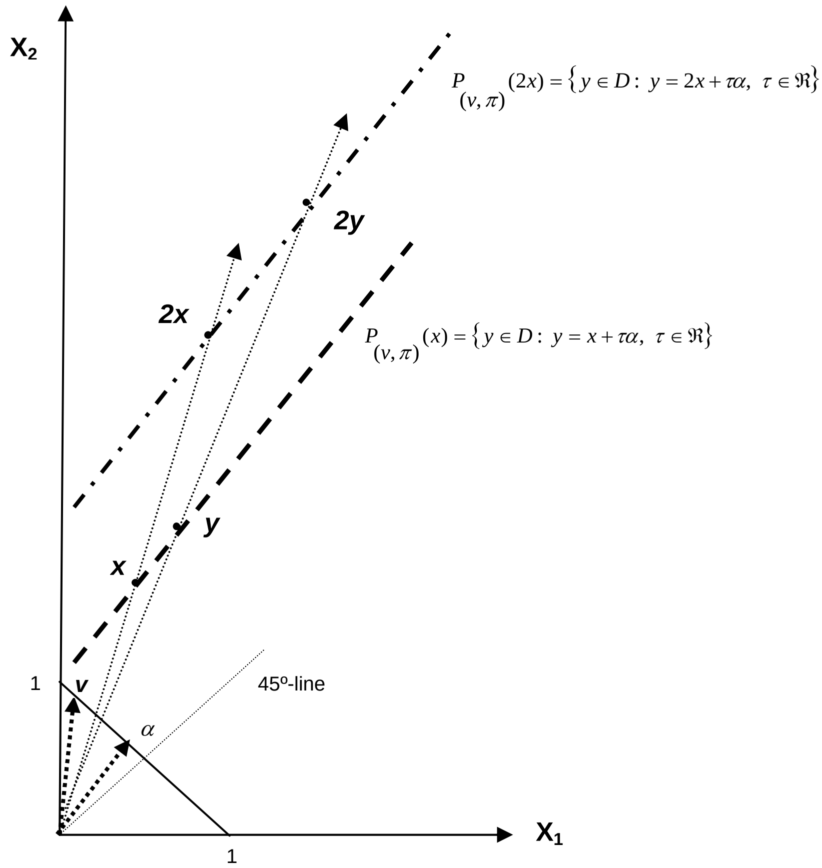

Figure 5. Invariance in DR-RC ($n = 2, \ \pi = 0.25$).

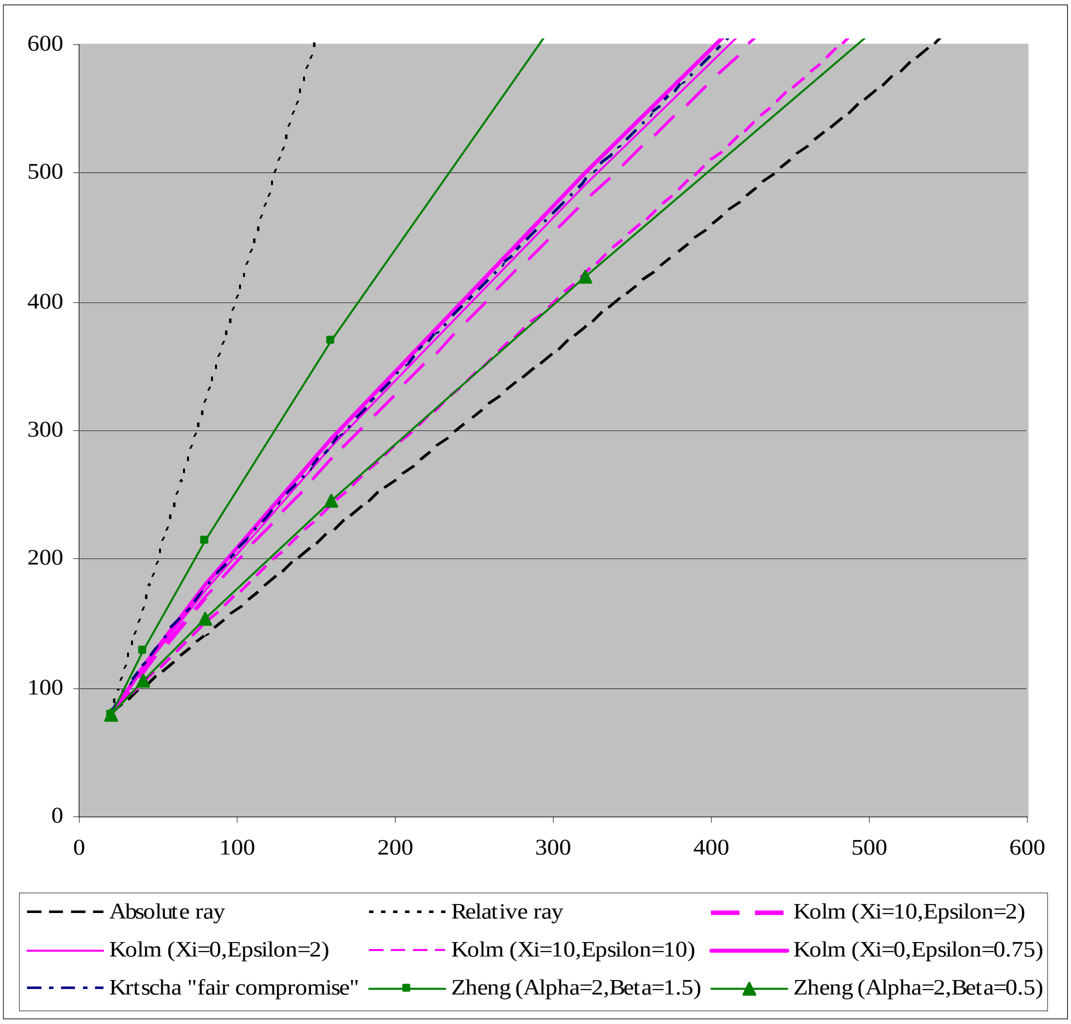

Figure 6. Iso-inequality contours corresponding to distribution $(20,80)$: Non-linear cases

Endnotes

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[1] Properties such as normalization, continuity, differentiability, and replication invariance are also commonly invoked, but they are of a more technical nature.

[2] Zheng (2007) characterized the entire class of unit-consistent decomposable inequality measures without imposing any invariance condition, so that relative, absolute, and intermediate measures are included in this class. Zheng (2005) also characterized theses measures when relaxing some assumptions (the differentiability assumption is replaced with continuity and decomposability is replaced with either aggregability or subgroup consistency).

[3] Zoli (2003) also proposed an analogous property, named “weak currency independence”, in a context of inequality equivalence criteria.

[4] The proposal by Zheng (2007) also includes extreme-rightist and extreme-leftist unit-consistent measures.

[5] See Besley and Preston (1988), Chakravarty (1988), Ebert and Moyes (2000), and Lambert (1993), among others.

[6] Note that the invariance line drawn in Figure 1 should actually finish at $x_1 = 0$, but we have enlarged it in order to make the picture clearer. The same considerations apply to Figure 2.

[7] This characteristic of the B-P invariance lines does not contradict the path independence axiom, since these lines do not intersect in the space of positive income distributions (the invariance lines cut at point $\left(-\frac{1-\mu}{\mu}, -\frac{1-\mu}{\mu}\right)$, as shown by B-P, see Figure 2). Therefore, the distribution reached after allocating extra income among individuals, without altering inequality, remains the same whether the extra income has taken place in only one step or in several, which allows building indices based on this notion. However, if two lines cut in the first quadrant, it would not be possible to construct inequality indices based on the invariance notion.

[8] This tendency to the relative ray was initially pointed by Seidl and Pfingsten (1997) and Del Río and Ruiz-Castillo (2000). More recently, Zheng (2004) offered a formal proof.

[9] This explains why Atkinson and Bradolini (2004) found similar empirical results either by using B-P's index or relative indices, even when considering extremely low μ – values $(\mu = 0.0027 \Leftrightarrow \xi = 365 \text{ dollars})$.

[10] We have changed their original notation in order to make it clearer. In particular, we have switched vector x by simplex vector v, since only the income shares of the distribution of reference are required to obtain the invariance ray.

[11] DR-RC can be considered as a special case of Seidl and Pfingsten (1997) since the latter previously proposed a α -invariance concept according to which any extra income should be distributed in fixed proportions, given by α, in order to keep inequality unaltered. Their vector does not have, however, a clear economic interpretation. On the other hand, their invariance concept does not satisfy horizontal equity, as discussed by Zoli (2003).

[12] Certainly, the same applies if comparing three or more distributions.

[13] Perhaps the notation employed by DR-RC led both Zoli (2003) and Zheng (2004) to the interpretation that v depends on distribution x, which is not the case. This confusion led to the former to conclude that DR-RC invariance notion does not satisfy the path independence axiom, when it really does (see Del Río and Alonso-Villar, 2007). It also led to the latter to conclude that DR-RC's proposal tends to an absolute notion when a given transformation that keeps inequality unaltered is performed repeatedly, while it does not. When the linear transformation $x + \tau\alpha$ is applied repeatedly to distribution x, its easy to see that in each iteration the difference between one distribution and the next one is equal to $\tau\alpha$ rather than τ.

[14] This method allowed Del Río and Ruiz-Castillo (2001) to compare income distributions in Spain between 1980 and 1990. By choosing v as the income shares corresponding to 1980's distribution, they concluded that for those people whose opinions are closer to the relative inequality notion (that is, if $\pi \in [0.87,\ 1]$), inequality would have decreased in Spain during that decade. However, for people more skewed towards the left side of the political spectrum (that is, if $\pi \in [0,\ 0.71]$), it would be the opposite.

[15] As in previous cases, Zheng (2004) proved that these curves become straight lines in the limit.

[16] Kolm's family of indices has iso-inequality contours that monotonically approach the absolute ray as either ξ or ε increases (if $\varepsilon > 1$). However, when $\varepsilon \in [0,1]$, there is no monotonicity with respect to this parameter. In this example, the contour closer to the relative ray ("rightist" view) is that corresponding to $\xi = 0, \varepsilon = 0.75$.

[17] Recent empirical evidence obtained by Atkinson and Brandolin (2004, p. 13) seems to support this idea: “Kolm’s centrist measure basically confirms the pattern shown by Kolm’s absolute measure”.

[18] Distribution w exists since distributions x and y are in the same plane (which is defined by the former and the egalitarian distribution), and the straight line representing distributions in that plane having a common mean $\overline{x}$, and the invariance line corresponding to distribution x always intersect.

[19] Another requirement is that one of the distributions has to belong to the intermediate space of the other (see definition of $\Gamma^{'}(\alpha)$ in section 2).